

# Modified Schrödinger dynamics with attractive densities

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## Abstract

The linear Schrödinger equation does not predict that macroscopic bodies should be located at one place only, or that the outcome of a measurement should be unique. Quantum mechanics textbooks generally solve the problem by introducing the projection postulate, which forces definite values to emerge during measurements; many other interpretations have also been proposed. Here, in the same spirit as the GRW and CSL theories, we modify the Schrödinger equation in a way that efficiently cancels macroscopic density fluctuations in space. Nevertheless, we do not assume a stochastic dynamics as in GRW or CSL theories. Instead, we propose a deterministic evolution that includes an attraction term towards the averaged density in space of the de Broglie-Bohm position of particles, and show that this is sufficient to ensure macroscopic uniqueness and compatibility with the Born rule. The state vector can then be seen as directly related to physical reality.

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Macroscopic uniqueness is not a natural physical consequence of standard quantum mechanics. This is because the linear Schrödinger equation can lead to situations where the position of macroscopic physical systems (the pointer of a measurement apparatus for instance) have non-zero probabilities to be at the same time at very different points of space. This difficulty is illustrated by the famous Schrödinger cat thought experiment: the linear evolution of the state vector leads to a state containing at the same time a dead and an alive cat. Schrödinger considers these superpositions of completely different macroscopic states as a “quite ridiculous case” [1, 2, 3]. The problem is that nothing in the dynamical equations can reduce the big fluctuations of the macroscopic density of particles that then occur. But macroscopic physical objects occupying simultaneously completely distinct positions have never been observed; when experiments are performed, a single position of the macroscopic measurement pointer seem to appear for each realization.

Many interpretations of quantum mechanics have been proposed to deal with this apparent contradiction. Historically, the projection postulate was introduced (but not approved by Bohr) by Heisenberg [4] and von Neumann [5], who started from an analysis of the measurement process in terms of quantum mechanics[6]. Von Neumann uses the Schrödinger equation to study the behavior of a chain of measurement apparatuses, and finds that no definite result will ever be obtained, even after a long chain of measurements

– this difficulty is known as the “infinite von Neumann regress”. He solves it by introducing his projection postulate, which assumes that a sudden change of the of the state vector is introduced, in order to update it with the information gained in a measurement. At the other extreme, in the Everett interpretation [7], the problem is solved by considering macroscopic uniqueness is not a physical phenomenon, but a delusion arising from the very functioning of the memory registers of human minds. Numerous other interpretations have been proposed [8]: modal, relational, consistent histories, informational, etc. Most of them do not change the standard equations of quantum mechanics, but focus on the best way to interpret the state vector, its relation with physical reality, information, experimental context, etc. All these interpretations are interesting, but for the moment none has emerged as the universally accepted optimal point of view.

Other families of interpretations consider that the problem of reconciling quantum dynamics with what seems to be a routine observation, namely the uniqueness of the classical world, should be taken seriously: the formalism and equations of quantum mechanics should be adapted to predict this uniqueness without ambiguity. The two best known categories are the de Broglie-Bohm (dBB) interpretation [9, 10, 11, 12] and the spontaneous localization theories, either in the original GRW discontinuous form [13], or in the continuous CSL form [14] – for a review, see for instance [15]. In the dBB theory, particle positions moving in ordinary 3D space are added to the variables of standard quantum mechanics; these positions have uncontrollable random initial values, but then move in a perfectly deterministic way. They are considered as physically real. In the GRW and CSL theories, no additional variables are assumed, but the usual Hamiltonian in the equation of motion of the state vector gets additional stochastic terms; the wave function is considered as a field propagating in configuration space (not ordinary 3D space) under the effect of fundamentally random processes. In these theories, measurement processes are not seen as special events, but just ordinary interaction processes between a measured system and apparatus; the observer is not a necessary ingredient of the theory.

Here we propose a combination of these two theories, where the dBB positions are still part of the dynamical equations: the positions are driven by the wave function (as in the dBB theory), but they also react on it (which does not occur in the dBB theory). The dynamics of this process is very different from that of GRW and CSL theories, since it is deterministic (no Wiener processes are assumed); the random character of a result of measurement is then just a consequence of the random value of the initial positions. The dynamics also suppresses the macroscopic “empty waves” of the dBB theory when they correspond to macroscopic systems (waves that never play a role in the future), eliminating any conceptual difficulty concerning the interpretation of these waves. It therefore seems to provide a simple and a reasonably plausible mechanism for quantum collapse.

## 1 Equations of motion with collapse

The change of the dynamics of the quantum state we propose is continuous (as opposed to the standard projection postulate), but nevertheless manages to suppress the cat paradox and the von Neumann’s infinite regress. It then becomes possible to consider, as in the

GRW and CSL theories [13, 14], that the state vector directly represents physical reality.

The study of Bohmian positions actually provides a convenient indicator of Schrödinger’s “ridiculous cases”, and therefore suggests a way to avoid them. Consider the Bohmian positions of the atoms contained in the glass bottle containing the poison that may kill the cat. After some time, the linear Schrödinger equation predicts a superposition of states where the bottle is broken and intact; the probability densities of the constituent atoms are spread between different locations in space. By contrast, for each realization of the experiment, the dBB theory predicts that the bottle is either broken or intact; the Bohmian positions of its constituents atoms remain bunched together in only one of the possible locations. This means that, in configuration space, one of the components of the quantum wave function propagates accompanied by Bohmian positions, while the other propagates “alone”; it has become what Bohm calls an “empty wave” [10]. In order to introduce macroscopic uniqueness in the propagation of the wave function, we will therefore introduce a dynamical process that suppresses the components that are propagating too far from the Bohmian positions; this will force a better match between the evolution of the wave function and that of the positions.

### 1.1 Bohmian localization operator $L$

We consider a system of  $N$  identical particles associated with a quantum field operator  $\Psi(\mathbf{r})$  defined at each point  $\mathbf{r}$  of ordinary 3D space. When the system is in state  $|\Phi\rangle$ , the local (number) density  $D_\Phi(\mathbf{r})$  of particles at  $\mathbf{r}$  is:

$$D_\Phi(\mathbf{r}) = \frac{\langle \Phi | \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}) | \Phi \rangle}{\langle \Phi | \Phi \rangle} \quad (1)$$

In dBB theory, the local density  $D_B(\mathbf{r})$  of Bohmian positions is a sum of delta functions:

$$D_B(\mathbf{r}, t) = \sum_{n=1}^N \delta(\mathbf{r} - \mathbf{q}_n) \quad (2)$$

where the sum runs over all  $N$  particles with Bohmian position  $\mathbf{q}_n(t)$ . We wish to introduce a dynamics that favors evolutions where  $D_\Phi(\mathbf{r})$  is attracted towards regions where  $D_B(\mathbf{r})$  is high, with a space average suppressing the microscopic fluctuations of  $D_B(\mathbf{r})$ . For this purpose, we introduce an averaging length  $a_L$  and the following integral of  $D_B$ :

$$N_B(\mathbf{r}, t) = \int d^3r' e^{-(\mathbf{r}-\mathbf{r}')^2/(a_L)^2} D_B(\mathbf{r}', t) = \sum_{n=1}^N e^{-(\mathbf{r}-\mathbf{q}_n)^2/(a_L)^2} \quad (3)$$

The order of magnitude of  $N_B(\mathbf{r}, t)$  is the number of Bohmian positions within a volume  $(a_L)^3$  around point  $\mathbf{r}$ ; we have  $0 \leq N_B(\mathbf{r}, t) \leq N$ . We then introduce the localization operator  $L(t)$  by:

$$L(t) = \int d^3r N_B(\mathbf{r}, t) \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}) = \sum_{p=1}^N N_B(\mathbf{R}_p, t) \quad (4)$$

( $\mathbf{R}_p$  is the position operator of particle  $p$ ). This operator combines the quantum density operator  $\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})$  with the classical averaged density  $N_B(\mathbf{r}, t)$ . It has the form of a single-particle potential energy operator;  $L(t)$  multiplies any wave function  $\Phi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  by the the sum over  $p$  of the individual potentials  $N_B(\mathbf{r}_p, t)$ .

The result depends on the relative positions of all Bohmian positions  $\mathbf{q}_n$ . When they are all at large relative distances from each other (larger than  $a_L$ ), the  $\mathbf{r}$  dependence of  $N_B$  exhibits a series of bumps centered on each  $\mathbf{q}_n$ , each of height unity, and separated by intervals where  $N_B$  practically vanishes; if none of the variables  $\mathbf{r}_p$  falls inside one of these bumps, the effect of  $L(t)$  is merely to cancel the wave function; if  $p$  of them fall inside these bumps, the effect of  $L(t)$  is roughly a multiplication by  $p$ . When, at the other extreme, all Bohmian positions are clustered together inside a single volume  $\mathcal{V}$  of size smaller than  $(a_L)^3$ , the  $\mathbf{r}$  dependence of  $N_B$  now exhibits a single bump of height  $N$ , and the effect of  $L(t)$  is more focussed in space; for instance, in the region of configuration space where all variables  $\mathbf{r}_p$  of  $\Phi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  fall inside  $\mathcal{V}$ , the wave function is multiplied by  $N^2$ . A more frequent case occurs when all positions are spread almost uniformly (at a scale  $a_L$ ) within a certain volume  $\mathcal{V}$ , as is for instance the case if the physical system is a piece of a solid containing a large number  $N_B$  of particles within a volume  $(a_L)^3$ . The localization effect then occurs inside volume  $\mathcal{V}$ , with a rate that is propostional to  $N \times N_B$ . Again we obtain a rate that is quadratic in the number of particles; this fast dependence plays an important role in the sudden collapse mechanism we discuss below.

## 1.2 Modified attractive quantum dynamics

We wish to introduce a dynamics that favors evolutions where  $D_\Phi(\mathbf{r})$  is attracted towards regions where  $N_B(\mathbf{r})$  is high. For this purpose, we add to the usual Hamiltonian  $H(t)$  a localization term proportional to  $L(t)$  and write the modified Schrödinger equation:

$$i\hbar \frac{d}{dt} |\Phi(t)\rangle = [H(t) + i\hbar\gamma_L L(t)] |\Phi(t)\rangle \quad (5)$$

where  $\gamma_L$  is a constant localization rate and  $a_L$  a localization length. The new term in the Hamiltonian increases the modulus of the wave function in regions where the Bohmian density is large. It is not Hermitian, and no longer conserves the norm of  $|\Phi\rangle$ . We consider that  $|\Phi\rangle$  defines the direction in the space of states (a one-dimension subspace of this space, what von Neumann calls a “ray”), so that its norm is irrelevant. Nevertheless, if desired, one can easily obtain a normalized state vector  $|\overline{\Phi}\rangle$ , which obeys the following equation of evolution:

$$i\hbar \frac{d}{dt} |\overline{\Phi}(t)\rangle = [H(t) + \overline{H}_L(t)] |\overline{\Phi}(t)\rangle \quad (6)$$

where:

$$\overline{H}_L(t) = i\hbar\gamma_L \int d^3r \left[ \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) - D_\Phi(\mathbf{r}) \right] N_B(\mathbf{r}, t) \quad (7)$$

The only difference with (5) is that the operator  $\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})$  has been replaced by that appearing inside the brackets, which is actually nothing but the operator associated with the fluctuation of the local density.

Equation (5) is linear but time-dependent, even if the Hamiltonian  $H$  is time independent, since the Bohmian positions and thus  $N_B(\mathbf{r}, t)$  depend on time. The norm-conserving version (7) is non-linear because  $D_\Phi(\mathbf{r}')$  depends on the state vector  $|\Phi\rangle$ . The modified dynamics we study in this article is defined by these time differential equations.

### 1.3 Coupled evolutions

The positions  $\mathbf{q}_n$  evolve according to the usual Bohmian equation of motion:

$$\frac{d\mathbf{q}_n(t)}{dt} = \hbar \frac{\vec{\nabla}_n \xi}{m} \quad (8)$$

where  $\xi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  is the phase of the wave function  $\Phi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$ . Since  $L(t)$  is diagonal and real in the position representation, it does not change the phase of the wave function, but only its modulus. The localization process therefore does not change the Bohmian velocities directly. It nevertheless changes them indirectly, because the evolution of the phase  $\xi(\mathbf{r})$  depends on the Laplacian of the modulus  $|\Psi|$  of the wave function:

$$\hbar \frac{\partial \xi}{\partial t} + \frac{\hbar^2}{2m} \left[ \sum_n \left( \left( \vec{\nabla}_n \xi \right)^2 - \frac{\Delta_n |\Psi|}{|\Psi|} \right) + V \right] \quad (9)$$

where  $\vec{\nabla}_n$  and Laplacian  $\Delta_n$  contain derivatives with respect to the 3 coordinates of particle  $n$ ;  $V$  is the usual potential operator. The term in  $\Delta_n |\Psi| / |\Psi|$  is often called the “quantum potential”. The effect of the localization process is to introduce smooth variations of  $|\Psi|$  taking place over a distance of the order of  $a_L$ . This spreads the Fourier components of the wave function over a range  $\Delta k \simeq 1/a_L$ ; only particles having de Broglie wavelengths  $\lambda$  that are of the order of (or larger than)  $a_0$  can undergo an appreciable change of their Bohmian velocity. With the mesoscopic value (10) chosen for  $a_L$ , this corresponds to very low velocities; they are transferred to the Bohmian positions at a rate  $\gamma_L$ , for which we will choose below a very small value. Altogether, after time integration, the localization terms produces very tiny changes of the Bohmian positions.

A general remark is that, in the limit  $a_L \rightarrow \infty$ , the localization term has no effect: in (3),  $N_B(\mathbf{r})$  then becomes equal to the number of particles  $N$  (a constant) and, in (4)  $L(t)$  becomes the products  $N\hat{N}$  (where  $\hat{N}$  is the operator associated with the total number of particles). The right hand side of (7) then becomes proportional to  $N(\hat{N} - N)$ , which gives zero when acting on any ket  $|\Phi(t)\rangle$  with a fixed number of particles; nothing is then changed with respect to standard Schrödinger dynamics.

## 2 Collapse in small or large systems

We now assume investigate the dynamics of physical systems obeying the modified Schrödinger equation (5). As in GRW and CSL theories, our purpose is to check that it is possible to find plausible values of the two parameters  $\gamma_L$  and  $a_L$ ; by this we mean values for which

no contradiction occurs with the enormous body of experimental data agreeing with quantum mechanics (sometimes with an incredible precision of  $10^{-12}$ !). What is needed is a compromise between opposite requirements: a localization dynamics that has fantastically small effects on microscopic systems, but nevertheless produces a sufficiently fast collapse of superpositions of macroscopically different states. We will choose values inspired by those often chosen in GRW and CSL theories, namely:

$$\begin{aligned}\gamma_L &= 10^{-16} \text{ s}^{-1} \\ a_L &\simeq 10^{-6} \text{ m}\end{aligned}\tag{10}$$

Our purpose here is not to define accurate values of these constants; we just wish to show that there is a wide range of values that are compatible with the above criteria.

## 2.1 Microscopic system

Consider first a microscopic system, atom, molecule or nucleus, with a wave function of all the constituent particles extending over a range  $a_0 \ll a_L$ . Since the  $\mathbf{q}_n$ 's can never reach regions of space where the wave function vanishes, they also remain localized in a region of space of dimension  $a_0$ . This corresponds to the case mentioned above, where  $N_B(\mathbf{r})$  is of the order of the total number of particles  $N$  in a domain of size  $a_L$  centered on the atom, and tends rapidly to zero outside of this domain. In the limit  $a_0/a_L \rightarrow 0$ , we have seen that  $L(t) \rightarrow N\hat{N}$ , so that in (5) the localization term has no effect on the wave function (except a multiplication by an overall factor without any physical consequence). If  $a_0/a_L \ll 1$ , the exponential in (3) can be approximated by  $1 - c(a_0/a_L)^2$ , where the term in 1 does not contribute (this is the limit  $a_L \rightarrow \infty$ ), and where  $c \simeq 1$  (the exact value of  $c$  depends on the Bohmian positions). So, retaining only the term in  $(a_0/a_L)^2$ , we see that the parts of the wave function at the periphery of the atom are reduced at a rate  $\gamma$  given by:

$$\gamma \lesssim \gamma_L \left( \frac{a_0}{a_L} \right)^2 N^2 \tag{11}$$

while the parts near the center of the atom remain unaffected.

For a small atom (Hydrogen or Helium for instance), with values (10),  $a_0/a_L \simeq 10^{-4}$  so that  $\gamma \leq 10^{-24} N^2$ , where  $N$  is a few units; this rate is clearly extremely low and undetectable. For a molecule, a size of 10 nm is already large, which corresponds to  $a_0/a_L \simeq 10^{-2}$  and to  $\gamma \leq 10^{-20} N^2$ ; even with a number of constituents (protons, neutrons) of the order of  $10^4$ , we still obtain an extremely small rate.

Now consider an interference experiment made with the same microscopic system. In the interferometer, its wave function is localized at the same time in very different regions of space; in one of these regions,  $N_B(\mathbf{r})$  is equal to  $N$  as above, but in the other it is zero. This clearly introduces an imbalance between the full wave, which increases at a rate  $\gamma_L N^2$ , and the (constant) empty wave. The rate of growth of this imbalance is:

$$\gamma \simeq \gamma_L N^2 \tag{12}$$

Therefore, even for a long experiment lasting one second, if  $N < 10^7$ , the localization rate remains negligible, and the interference takes place as in standard Schrödinger dynamics; but, for larger values of  $N$ , this dynamics predicts that the contrast of fringes should decrease and vanish in the limit  $N \gg 1/\sqrt{\gamma_L t}$ .

## 2.2 Macroscopic system

The orders of magnitude are completely different for macroscopic systems. Consider for instance the pointer of a measurement apparatus; after measurement it may reach (for instance) two different positions that are 10 microns apart from each other. The solution of the linear Schrödinger equation has components where the particles of the pointer are, either in one region of space, or in another; big fluctuations of the local density of particles then take place. By contrast, in a given realization of the experiment, the corresponding Bohmian variables remain all clustered in the vicinity of only one of these two positions. They necessarily remain together because the state vector has no components where some of the pointer particles are in one site, some in the other: this is forbidden by the cohesion forces inside the material forming the pointer. So, when the measurement is performed, one component of the wave function resides in the same region of configuration space as many Bohmian variables (this is a “full wave”), but the other component in region where the density of Bohmian positions is zero (this is an empty wave).

For the “full wave”, in the localization term in the right hand side of (5), the relevant values of  $\mathbf{r}$  in the integral are those in the region of space where this wave propagates. Since  $N_B$  has significant values in this region, this term increases the modulus of the full wave. We have seen that  $L(t)$  is a potential operator, the sum of  $N$  individual potentials that are equal to  $N_B$ ; its effect on the wave function is to multiply it by the product  $N_B N_P$ , where  $N_P$  is of the order of the number of particles in the pointer and  $N_B$ , the number of its particles in volume  $(a_L)^3$ . As for the “empty wave”, the relevant values of  $\mathbf{r}$  are those in the region of space where  $N_B$  is zero, and the localization term does not have any effect. When the measurement result is fully registered in the position of the particles of the pointer, the relative weight of the full wave is therefore increased exponentially with a time rate  $\gamma$  of the order of:

$$\gamma \simeq \gamma_L N_B N_P \quad (13)$$

If we choose conservatively small values  $N_P = 10^{20}$ ,  $N_B = 10^{11}$ , we obtain a very fast rate  $\gamma \simeq 10^{15}$ ; the dynamical equation leads to an extremely fast collapse of the wave function!

After collapse, the state vector of the physical system continues to evolve under the influence of the Hamiltonian  $H(t)$  and of the localization term. Their effects are very different, since the full Hamiltonian  $H(t)$  contains interactions between the particles and directly controls the correlations between the particles, while the localization term is just a single particle operator, similar to mean field operator (with an anti-Hermitian contribution to the evolution). In a low compressibility solid or liquid, the average energy  $\langle H \rangle$  varies very rapidly as a function of the average distance between the particles; the weak localization term cannot change  $\langle H \rangle$  significantly, so that the quantum average distance between the particles remains practically constant, leading to a density  $D_\Phi(\mathbf{r})$  that is almost uniform in the volume of the solid.

We now examine how the ensemble of  $\mathbf{q}_n$ 's reacts to this wave function in configuration space. We have mentioned above that the collapse process may affect the  $\mathbf{q}_n$ 's indirectly. Nevertheless, Towler, Russell and Valentini [16] have shown that a fast relaxation process tends to constantly bring back any distribution of the  $\mathbf{q}_n$ 's in configuration space towards that of quantum equilibrium<sup>1</sup> (the coarse graining considered in this reference is immediately provided here by the average over the localization length  $a_L$ ); the distribution of the  $\mathbf{q}_n$ 's then closely follows the square of the wave function. As a consequence, in ordinary space, a spatially uniform  $D_\Phi(\mathbf{r})$  results in a practically uniform distribution of  $N_B(\mathbf{r})$ . The only effect of the term in  $L(t)$  is then to localize all particles within the volume of the whole solid – but this is nothing but what is usually done in many-body physics, when one assumes that the physical system is contained in a box. The standard Schrödinger dynamics therefore applies with no change.

### 2.3 Bose-Einstein condensate

Consider now a gaseous Bose-Einstein condensate that is partly reflected by Bragg scattering on a laser standing wave [17]. A matter wave is then split into two coherent parts, which can propagate at macroscopic distances and interfere again if recombined. We must check that the localization term does not destroy the coherence, which would be in contradiction with the experimental observations. The major difference with the preceding case is that the atoms in the condensate propagate almost freely, and that no process forces all of them to go in the same direction; there is no reason to find all atoms in the same output beam. The distribution of Bohmian positions then closely follow the quantum distribution (a Poisson distribution of populations in the two output atomic clouds). In other words, none of the two waves becomes empty; each of them travels accompanied by a Bohmian density that is proportional to its intensity. Moreover, the number of atoms involved in these experiments is of the order of  $10^5$ , in a volume that is comparable with  $a_L$ . So, even in the absence of Bohmian density, relation (13) would lead to  $\gamma \simeq 10^{-6}$ , still a very small rate. The collapse predicted in the previous case does not take place here, and the two waves propagate as coherent classical waves.

### 2.4 Measurement, Born rule

As soon as a microscopic quantum system  $S$  interacts with a measurement apparatus  $M$ , it becomes entangled with some of its particles: each measurement eigenstate  $|s_i\rangle$  of  $S$  becomes associated with different states of  $M$ . As we have seen, the usual Schrödinger dynamics is not affected until a large number  $N_m$  of particles of  $M$  is involved in this process. But, since the very purpose of a measurement apparatus is to transfer information to a macroscopic scale, entanglement progresses rapidly within the measurement apparatus (it propagates by “contagion” between mutually interacting neighbor particles, as discussed

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<sup>1</sup>These authors study the evolution of the distribution of the  $\mathbf{q}_n$ 's when the wave function obeys the standard Schrödinger equation (without any localization term). Since they predict relatively short relaxation times, their conclusions for microscopic objects should not be changed significantly by the introduction of a localization term having a very small coupling constant.



in [18]); it quickly reaches a mesoscopic and macroscopic scale. In standard quantum mechanics, this phenomenon occurs in parallel in all “measurement channels” (values of the index  $i$ ); various branches of the state vector develop, each associated with one of the measurement eigenstates  $|s_i\rangle$ . In Bohmian mechanics, this propagation induces a motion of the Bohmian positions; nevertheless, for each realization of the experiment, the motion occurs only within one measurement channel (all the other channels correspond to empty waves having no effect on the Bohmian positions). The changes of the values of  $N_B(\mathbf{r}, t)$  are therefore those associated with a single branch of the state vector. In other words, for each realization of the measurement, the time dependence of  $N_B(\mathbf{r}, t)$  is different and, in (5), the operator  $L(t)$  tends to localize the wave function in a different way.

Consider now a time when the values of  $N_B(\mathbf{r}, t)$  have become significantly different for each value of  $i$  in macroscopic regions of space; by the process discussed above for macroscopic systems, the effect of the localization process on the wave function is then very fast, and selects in the wave function the component associated with a single value of  $i$  (one result of measurement). State vector reduction has then taken place. As a whole, the localization process is similar to fast freezing of a liquid: the suppression of the “empty” components of the wave function occurs with a time constant that is initially completely negligible, but grows faster than linearly in time (exponentially at the beginning of propagation of entanglement).

Now, instead of a single realization of the experiment, consider an ensemble of realizations. When the initial Bohmian positions of the particles are chosen randomly according to the quantum distribution, the dBB theory exactly reproduces the Born rule of standard quantum mechanics. As a consequence, before the sudden localization process of the wave function takes place, the effects of the localization term on the positions are still negligible, and the standard quantum probability gives the proportion of realizations where the  $N_m$  Bohmian positions of the measurement apparatus reach the region of space associated with a measurement eigenstate  $|s_i\rangle$ . For the wave function, this region determines the branch that will be enhanced by the localization process, while all the empty waves are cancelled. As a consequence, the collapse of the wave function takes place randomly towards one of the regions of space associated with the various possible results of measurement, with a probability given by the standard Born rule. Without conflict with the predictions of quantum mechanics, we can then consider the field associated with  $|\overline{\Phi}(t)\rangle$  (within a phase factor) as directly related to physical reality (in configuration space).

## 2.5 Effects of decoherence, stabilization

Decoherence plays a very important role in standard quantum mechanics [19], even if it does not ensure macroscopic uniqueness. The dynamics resulting from (5) provides this uniqueness: even if no measurement is performed, the interactions of the physical system with its environment produce collapse through the localization mechanism. To illustrate this with an example, let us consider a system of 3 particles, two with Bohmian positions at a small relative distance (smaller than  $a_L$ ; we can then consider that these positions coincide:  $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{u}$ ), and a third far away ( $\mathbf{q}_3 = \mathbf{v} \neq \mathbf{u}$ ). We assume that  $|\varphi_u^0\rangle$  is an individual state localized around  $\mathbf{q}_1 = \mathbf{q}_2$  and that  $|\varphi_v\rangle$  is another individual state

localized around  $\mathbf{q}_3$ . The initial quantum state of the 3 particles is a superposition:

$$\begin{aligned} |\Phi(0)\rangle = & \alpha |1 : \varphi_u^0\rangle |2 : \varphi_u^0\rangle |3 : \varphi_u^0\rangle |E_u^0\rangle |E_v^0\rangle \\ & + \beta |1 : \varphi_u^0\rangle |2 : \varphi_u^0\rangle |3 : \varphi_v^0\rangle |E_u^0\rangle |E_v^0\rangle \end{aligned} \quad (14)$$

where  $\alpha$  and  $\beta$  are arbitrary complex coefficients; the kets  $|E_u^0\rangle$  and  $|E_v^0\rangle$  denote the initial states of the environments surrounding the two regions of space of  $u$  and  $v$ . Note that the first component of the superposition contains a mismatch between the localization of the particles in the state vector (the three at the same site) and their Bohmian positions (only two particles have the same Bohmian position). If the system remained totally isolated, this mismatch could continue to exist forever, especially since the first component is favored by a larger local value of  $N_B$ .

But this does not happen in practice, since a physical system always interacts with its environment. When the particles become locally entangled with their environment, the state vector  $|\Phi(t)\rangle$  becomes:

$$\begin{aligned} |\Phi(t)\rangle = & \sum_{i,j,k} \alpha'_{i,j,k} |1 : \varphi_u^i\rangle |2 : \varphi_u^j\rangle |3 : \varphi_u^k\rangle |E_u^{i,j,k}\rangle |E_v^0\rangle \\ & + \sum_{l,m,n} \beta'_{l,m,n} |1 : \varphi_u^l\rangle |2 : \varphi_u^m\rangle |3 : \varphi_v^n\rangle |E_u^{l,m,n}\rangle |E_v^{l,m,n}\rangle \end{aligned} \quad (15)$$

where the  $\alpha'_{i,j,k}$  and  $\beta'_{l,m,n}$  are the coefficients characterizing these local entanglements; the states where the upper index 0 has been replaced by other indices denote the states taking part in this entanglement. In the first line, the state  $|E_v^0\rangle$  of the environment at  $v$  remains unchanged, since for this component the wave function contains no particle in this site; as for the state  $|E_u^0\rangle$  of the environment around  $u$ , it is changed in different ways, depending if this environment interacts with 2 or 3 particles. Now, the Bohmian velocities are obtained from the phase of the wave function at the point of configuration space defined by all Bohmian positions; since we have assumed that one particle is in site  $v$ , only the phase of the components in the second line are relevant to this calculation (all components of the first line are empty waves). As a consequence, while the states of the environment in the second line propagate in space accompanied by Bohmian positions, the states in the first line do not. While these positions move in the environment, the localization term then favors the components of the second line, exponentially reducing the relative weight of those in the first line. If the environment is macroscopic, the corresponding collapse is very fast; in practice, only the components where the positions of particles are close to the Bohmian positions survive. Through this process, the environment has a stabilizing effect of the wave function towards the Bohmian trajectories of all particles; it rapidly corrects any mismatch between the propagation of the state vector and that of Bohmian positions.

### 3 Discussion

The localization process contained in equation (5) occurs in ordinary 3D space, as in CSL theory, but is also very different. CSL introduces the simultaneous action of an infinite

number of random processes modifying the wave function in all points of space, and described by Wiener processes. A “probability rule” ensures that the processes cooperate in space in order to conserve the maximum possible norm to the state vector, which leads to the usual Born rule. Gisin [20] also assumes the presence of an additional random term in the Schrödinger equation; the evolution of the state vector depends on a stochastic Wiener process which, conversely, has an evolution that depends on the state vector.

Here, no stochastic process is introduced, and the evolution is entirely deterministic. Randomness arises only from the initial distribution of the dBB positions. The time dependence of the localization term equation arises from the time dependence of the Bohmian density  $N_B(\mathbf{r}, t)$ , which is different in different realizations of the same experiment. If no important fluctuation of the Bohmian density occurs from one realization to another, the localization term remains completely negligible; but, in measurement-like situations, it quickly collapses the wave function.

Another difference with GRW and CSL theories is that the localization process does not take place with a constant localization length. It occurs over a variable range that depends on the space distribution of matter; for instance, no localization happens anymore when the density of the system becomes perfectly uniform. In a solid, as we discussed at the end of § 2.2, the localization takes place inside a volume that is nothing but the whole volume of the solid. As a consequence, the effects of the added term in the evolution can be much softer than in standard localization theories. It corresponds to a much smaller spontaneous heating of material bodies than GRW and CSL – but it nevertheless predicts the disappearance of interferences for macroscopic objects roughly in the same situations as these theories. Another difference is that the stochastic processes of GRW and CSL are Markovian processes, with no memory; here, the Bohmian variables introduce some memory, since their positions depend on previous velocities, and therefore of previous values of the wave function. As a consequence, the simple Lindblad forms for the evolution of the density operator in GRW and CSL theories are no longer obtained.

The equations obey Galilean relativity (absolute time), which is logically consistent for a modification of the Schrödinger equation; it would nevertheless be interesting to extend them to Einsteinian relativity, for instance with a time delayed density localization term.

Allori et al. [21] have proposed an interesting related idea: multiply the usual wave function in configuration space by a function centered around all individual Bohmian positions; this truncation provides a “collapsed” wave function. The two wave functions are directly related at any time  $t$ ; they do not have really independent evolutions. With this dynamics, the collapsed waves that have been truncated may reappear later to create interference; the new wave function is no longer symmetrical by exchange of identical particles. This scheme depends only on one parameter (the localization length  $a_L$ ) instead of two in our case; also, the collapse is performed in configuration instead of real space. Bedingham [22] has also combined Bohmian mechanics with CSL by introducing a stochastic term into the Schrödinger equation that depends on the positions of the particles, as well as non-linear filtering techniques. Tumulka [23] has proposed a simpler version of a stochastic theory that is basically the same.

**Conclusion:** The equations of evolution proposed here imply that, in practice, no significant collapse occurs unless a macroscopic fluctuation of density exists in ordinary

space, meaning that entanglement must have propagated from a microscopic to a macroscopic level. This is precisely, by construction, what a measurement apparatus is supposed to do with a microscopic system. Collapse takes place when quantum macroscopic density fluctuations occur within the pointer of a measurement apparatuses, but neither measurements nor observers play any special role.

Different attitudes are possible to interpret such dynamical equations. One is to consider that the Bohmian positions are the essence of reality, as in the dBB theory. The advantage of the proposed dynamics is then to get rid of the macroscopic “empty waves”, which persist forever in the dBB theory, while they are supposed to play no physical role whatsoever; their existence complicates the discussion of the physical reality of the waves. Another attitude is to consider that the directly observable component of physical reality is the field (the wave function); the other component of reality would then be the attractive density  $N_B(\mathbf{r})$ , acting on the field but not directly observable (a sort of “dark density”). Bohmian positions may then be considered as the tool to generate the propagation in real space of an attractive density  $N_B(\mathbf{r})$ ; this is slightly reminiscent of de Broglie’s ideas on singularities associated with the wave function and propagating with it (theory of the double solution). The propagation in space of the field is then free of the counter-intuitive aspects attached to Bohmian positions (changing direction in free space for instance). Nevertheless, in this view, a naive realism is not restored: the field remains very different from a classical field, since it does not propagate in ordinary 3D space.

The main purpose of the present work can be seen as a proof of existence: as GRW and CSL theories have already shown, it is possible to build a simple theory where waves represent physical reality, while remaining compatible with present experimental data. Our contribution is to show that introducing a stochastic dynamics is not a necessary condition. Macrorealism [24] can indeed emerge from the dynamics, without elaborate mathematics in the equations. Needless to say, it remains perfectly legitimate to invoke esthetical or philosophical reasons to maintain the Schrödinger dynamics unchanged, and adhere to one of the interpretations that are available. But one can also prefer to change the dynamics to obtain a completely unified dynamics.

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